Grade Level/Course: **Algebra 1 or Algebra 2**
Lesson/Unit Plan Name: **Writing Exponential Functions Based on Data—Introductory Lesson**
Rationale/Lesson Abstract: This lesson is a hands-on introduction to exponential functions using a paper folding activity. Students collect data from the paper folding activity, describe the patterns, graph the data and write a function based on the pattern. The lesson then asks students to compare linear and exponential sequences. Finally, students work on an application problem that addresses exponential growth. Prerequisite skills needed include: • Writing a rule in functional notation • Graphing coordinates from a table of data. • Writing exponential expressions in expanded form (decomposition)
Timeframe: **1 period ~ 50 minutes**
Common Core Standard(s): **Building Functions--F.BF.1**—Write a function that describes a relationship between two quantities. **Linear, Quadratic and Exponential Models--F.LE.2**--Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Instructional Resources/Materials:

- Handout (provided)
- Blank sheets of paper—Could be two per student, two per pair or two per group.
- Exit tickets (provided)

Activity/Lesson:

Warm Up:

A. Decompose 16 to its prime factors.

B. Expand and simplify:

$$\left(\frac{1}{2}\right)^3 =$$

After distributing the handout and blank sheets, ask students to read the title and the introductory paragraph. Tell students that the paper folding activity is divided into two parts, one for each blank paper they have. Ask students to find numbers 1 through 5 in Part I to familiarize themselves with the sequence of steps they will take.

As a class, model the first two folds of the paper and fill out the first few rows of the table. Ask students to complete Part I together. Do a quick debrief after 5-7 minutes. Then ask students to read the directions for Part 2 aloud in their team and to begin that section.

After about 10 minutes, do a quick debrief and move on to Part 3 and do the We Do's together as a class. Ask students to do the You Try's with their team and to complete Part 3. This should take about 10 minutes. The primary understanding here is that linear sequences feature repeated addition, while exponential sequences feature repeated multiplication. Post or debrief quickly the solutions.

Students should then complete Part 4 as a team. Post the solutions and provide a quick debrief.

As a class, provide students with the generalized form of an exponential function and a linear function, pointing out variables for initial conditions and growth rate/factor.

Assessment: Hand out the exit tickets and allow 2-3 minutes to complete. Collect and determine success of your lesson based on percentage of acceptable answers.

Today you will: 1. Find patterns that lead to exponential growth, 2. Graph these patterns, 3. Describe the pattern as a rule and 4. See the difference between linear and exponential patterns.

☺ The Paper Folding Activity!! ☺

Part 1: Number of Sections

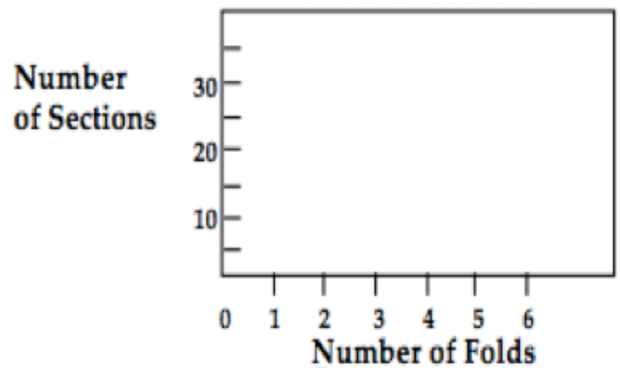
1. Fold a sheet of paper in half, reopen the sheet and count the number of sections the paper now has.

2. Record this data in the table and continue folding until it's impossible to fold anymore! ☺

Number of Sections

Number of Folds f	Number of Sections $S(f)$	*...Number of Sections as a power of 2..*
0		2^0
1		2^1
2		
3		
4		
5		
6		

3. Plot your data on the graph:



4. Describe the pattern (rule) that you see in the table.

"Each fold will…

5. Write a mathematical sentence (equation) that would represent this **exponential** (hint hint) growth.

$S(f) =$

Part 2: Area of Smallest Section

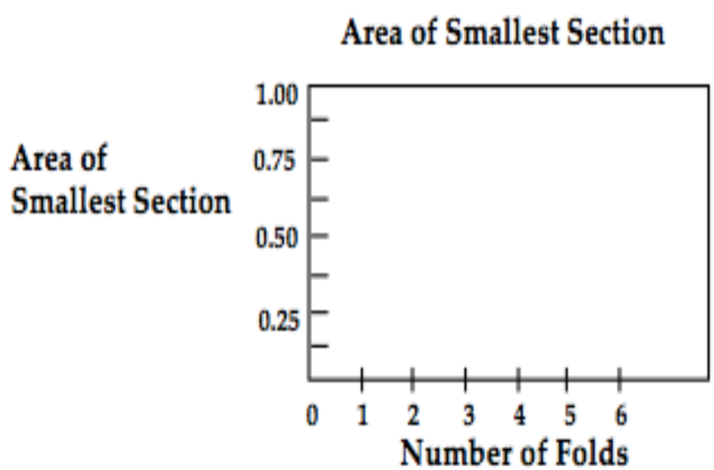
1. Fold a sheet of paper in half, reopen the sheet and determine the AREA of the smallest section the paper now has.

2. Record this data in the table and continue folding until it's impossible to fold anymore! ☺

Area of Smallest Section

Number of Folds f	Area of Smallest Section $A(f)$	Area of Smallest Section *A(f) as a power of ½*
0	1	$\left(\frac{1}{2}\right)^0$
1		$\left(\frac{1}{2}\right)^1$
2		
3		
4		
5		
6		

3. Plot your data on the graph:



4. Describe the pattern (rule) that you see in the table.
"Each fold will

5. Write a mathematical sentence (equation) that would represent this **exponential** decay.

$A(f) =$

Part 3: Find and describe the sequence.

We Do….	You Try…	
20, 15, 10, ____, ____, _____... Describe:	2, 4, 6, 8, ____, ____, _____... Describe:	3, 4.5, 6, 7.5, ____, _____... Describe:
8, 4, 2, 1, ____, ____, ______... Describe:	2, 8, 32, 128, ____, _____ … Describe:	1, 3, 9, 27, ____, _____... Describe:

A **linear growth** rate of change shows repeated ______________________,

where an **exponential growth** rate show repeated _____________________.

Make your own linear growth sequence!	Make your own exponential growth sequence!

On a graph, linear growth looks like a straight line. Exponential growth looks like _________________.

Part 4: Application Problem

The population of bacteria in a petri dish **doubles** every hour. If there are initially 3 bacteria in a dish, how many bacteria will there be in 5 hours? Show this growth in a table, graph and equation.

Hours (h)	0	1				h
# of bacteria after h hours: b(h)	3					$b(h)$
Numerical decomposition of number of bacteria				$3\cdot2\cdot2\cdot2$ or $3\cdot2^3$		

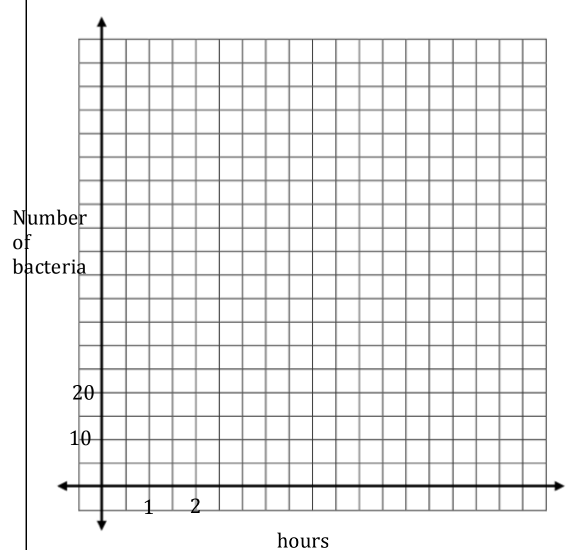
At this rate, there will be __________bacteria after 5 hours.

Initial condition:

Growth rate:

Equation:

Petri Dish Population



The equation we discovered above can be generalized:	Compare this to one of our linear equations:
Exponential Growth:	Linear Growth:

Exit Ticket Name:________________________ Give examples of a linear growth pattern and an exponential growth pattern. Explain how they are different.	Exit Ticket Name:________________________ Give examples of a linear growth pattern and an exponential growth pattern. Explain how they are different.
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MA#_____: Writing Exponential Functions Based on Data **Name:____________________**

Today you will: 1. Find patterns that lead to exponential growth, 2. Graph these patterns, 3. Describe the pattern as a rule and 4. See the difference between linear and exponential patterns.

☺ The Paper Folding Activity!! ☺

Part 1: Number of Sections

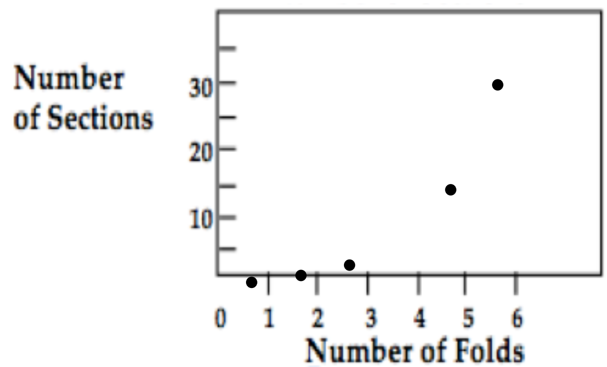
1. Fold a sheet of paper in half, reopen the sheet and count the number of sections the paper now has.

2. Record this data in the table and continue folding until it's impossible to fold anymore! ☺

Number of Sections

Number of Folds f	Number of Sections $S(f)$	*...Number of Sections as a power of 2..*
0	**1**	2^0
1	**2**	2^1
2	**4**	2^2
3	**8**	2^3
4	**16**	2^4
5	**32**	2^5
6	**64**	2^6

3. Plot your data on the graph:



4. Describe the pattern (rule) that you see in the table.

"Each fold will… **(answers will vary).**

5. Write a mathematical sentence (equation) that would represent this **exponential** (hint hint) growth.

$$S(f) = 2^f$$

Part 2: Area of Smallest Section

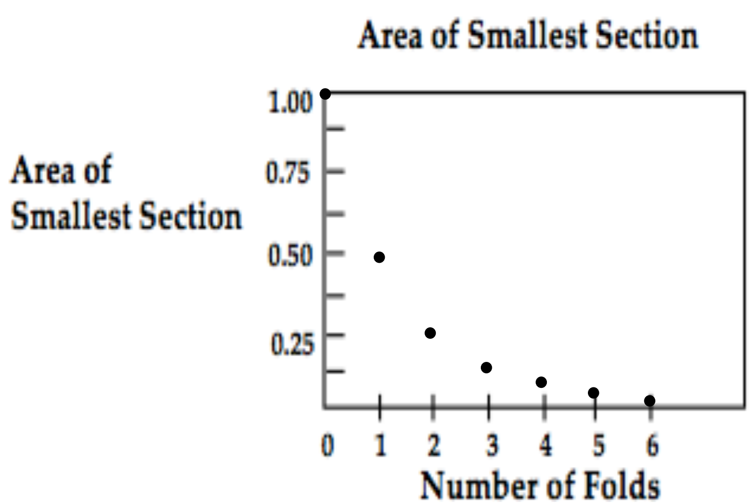
1. Fold a sheet of paper in half, reopen the sheet and determine the AREA of the smallest section the paper now has.

2. Record this data in the table and continue folding until it's impossible to fold anymore! ☺

Area of Smallest Section

Number of Folds f	Area of Smallest Section $A(f)$	Area of Smallest Section *$A(f)$ as a power of ½*
0	1	$\left(\frac{1}{2}\right)^0$
1	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
2	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
3	$\frac{1}{8}$	$\left(\frac{1}{2}\right)^3$
4	$\frac{1}{16}$	$\left(\frac{1}{2}\right)^4$
5	$\frac{1}{32}$	$\left(\frac{1}{2}\right)^5$
6	$\frac{1}{64}$	$\left(\frac{1}{2}\right)^6$

3. Plot your data on the graph:



4. Describe the pattern (rule) that you see in the table. "Each fold will… (answers will vary)	5. Write a mathematical sentence (equation) that would represent this **exponential** decay. $A(f) = \left(\frac{1}{2}\right)^f$

Part 3: Find and describe the sequence.

We Do….	You Try…	
20, 15, 10, **5, 0, -5** Describe: **subtracting 5 (or adding -5)--Linear**	2, 4, 6, 8, **10, 12, 14** Describe: **adding 2--Linear**	3, 4.5, 6, 7.5, **9, 10.5, 12** Describe: **adding 1.5--Linear**
8, 4, 2, 1, **½, ¼, 1/8** Describe: **dividing by 2 (or multiplying by ½)--Exponential**	2, 8, 32, 128, **512, 2048** Describe: **Multiplying by 4--Exponential**	1, 3, 9, 27, **81, 243** Describe: **Multiplying by 3--Exponential**

A **linear growth** rate of change shows repeated **ADDITION**,

where an **exponential growth** rate show repeated **MULTIPLICATION.**

Make your own linear growth sequence! ANSWERS WILL VARY	Make your own exponential growth sequence! ANSWERS WILL VARY

On a graph, linear growth looks like a straight line. Exponential growth looks like **A CURVE.**

Part 4: Application Problem

The population of bacteria in a petri dish **doubles** every hour. If there are initially 3 bacteria in a dish, how many bacteria will there be in 5 hours? Show this growth in a table, graph and equation.

Hours (h)	0	1	2	3	4	h
# of bacteria after h hours: $b(h)$	3	6	12	24	48	$b(h)$
Numerical decomposition of number of bacteria	3 or $3\cdot2^0$	$3\cdot2$ or $3\cdot2^1$	$3\cdot2\cdot2$ or $3\cdot2^2$	$3\cdot2\cdot2\cdot2$ or $3\cdot2^3$	$3\cdot2\cdot2\cdot2\cdot2$ or $3\cdot2^4$	$3\cdot2^h$

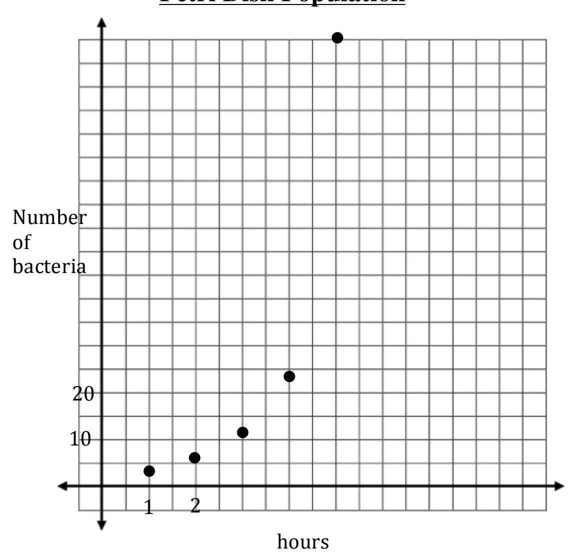
At this rate, there will be 96 bacteria after 5 hours.

Initial condition: 3

Growth rate: times 2

Equation: $b(h) = 3\cdot2^h$

Petri Dish Population



The equation we discovered above can be generalized: Exponential Growth: $f(t) = a\cdot b^t$ where a is the initial condition (starting value), b is the growth factor, and t is time.	Compare this to one of our linear equations: Linear Growth: $f(x) = mx + b$ where m is the growth rate and b is the initial condition (starting value).